Physics 101

Oscillatory Motion

Lecture 5

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***Periodic motion** is motion of an object that regularly repeats the object returns to a* **given position after a fixed time interval**

- ***Several types of periodic motion in everyday life.***
- ***The Earth returns to the same position in its orbit around the Sun each year.***
- ***The Moon returns to the same relationship with the Earth and the Sun, resulting in a full Moon approximately once a month.***
- ***In addition to these everyday examples, For example,***
- ***the molecules in a solid oscillate about their equilibrium positions;***
- ***Electromagnetic waves,such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors;***
- ***and in alternating-current electrical circuits, voltage, current, and electric charge vary periodically with time.***
- ***A special kind of periodic motion occurs in mechanical systems when the force acting on an Object is proportional to the position of the object relative to some equilibrium position.***
- ***If this Force is always directed toward the equilibrium position, the motion is called simple Harmonic motion, which is the primary focus of this chapter.***

Simple Harmonic or Periodic Motion

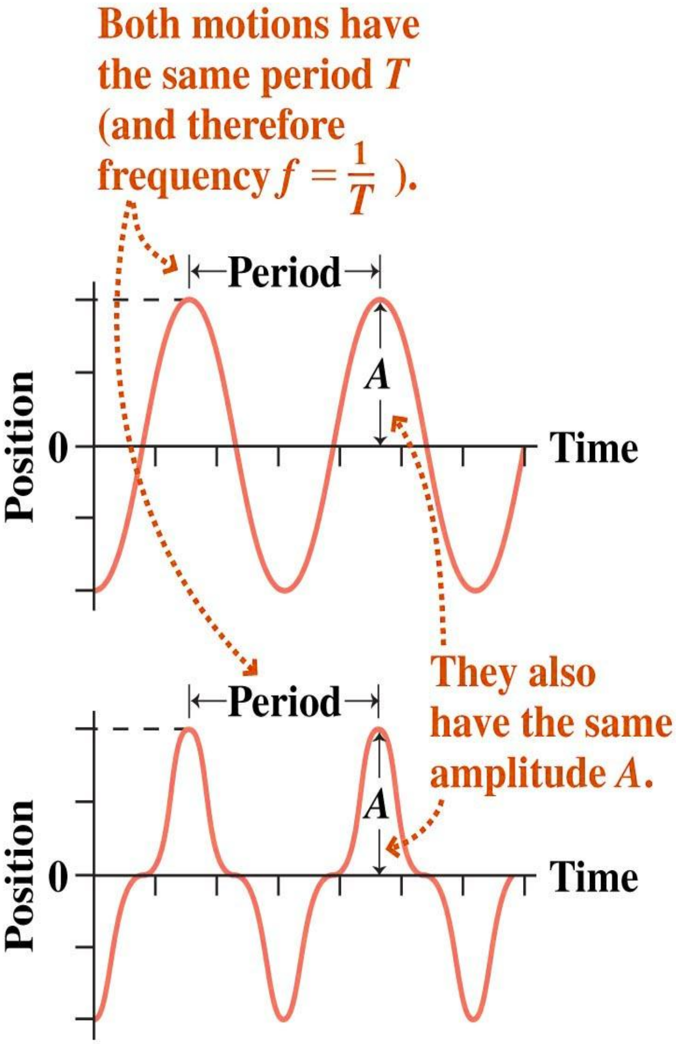
1- **Any motion that repeats itself in equal intervals of time is called:**

periodic motion or Simple harmonic motion

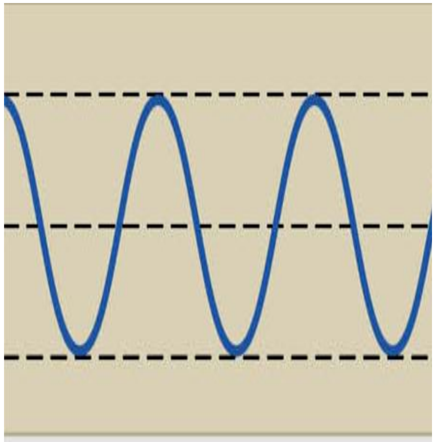
2- **If a particle in periodic motion move back and forth about its equilibrium, that is, it will *oscillate* and the motion is called:**

oscillatory or vibratory motion.

. The maximum displacement From the equilibrium is called the amplitude, *A*.



The period and frequency of a wave



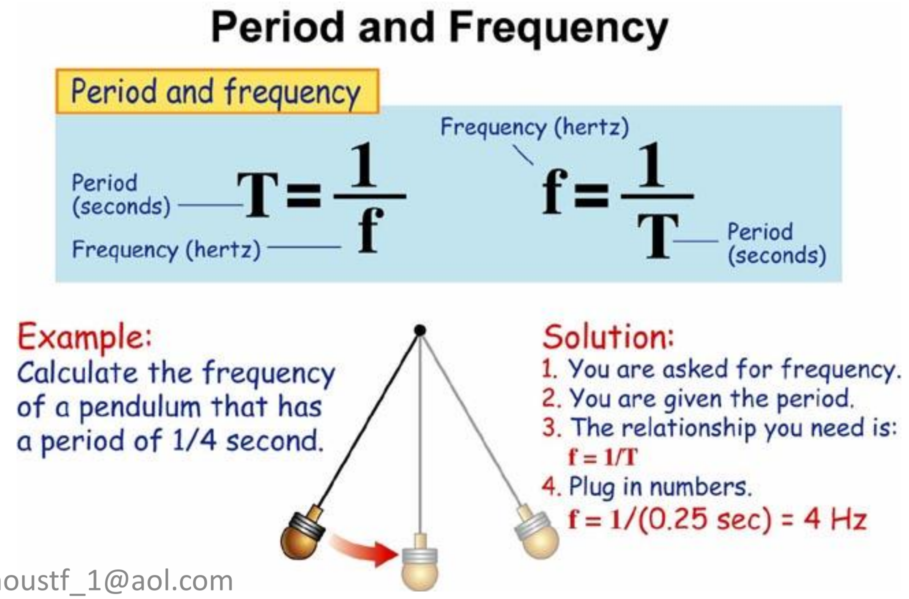
- the period T of a wave is the amount of time required to complete one cycle
- the frequency v is the number of cycles per second
  - the unit of a cycle-per-second is commonly referred to as a hertz (Hz), after Heinrich Hertz (1847-1894), who discovered radio waves.

$$v = \frac{1}{T}$$

- frequency and period are related as follows:

$$\omega = 2\pi f$$

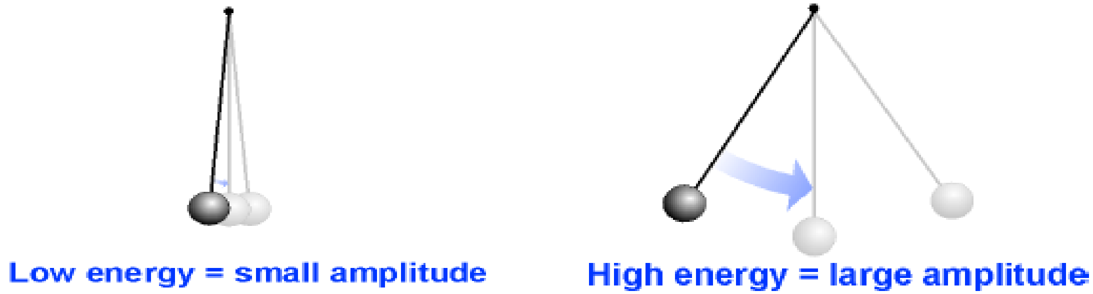
- Relationship between frequency and angular frequency is:



Amplitude

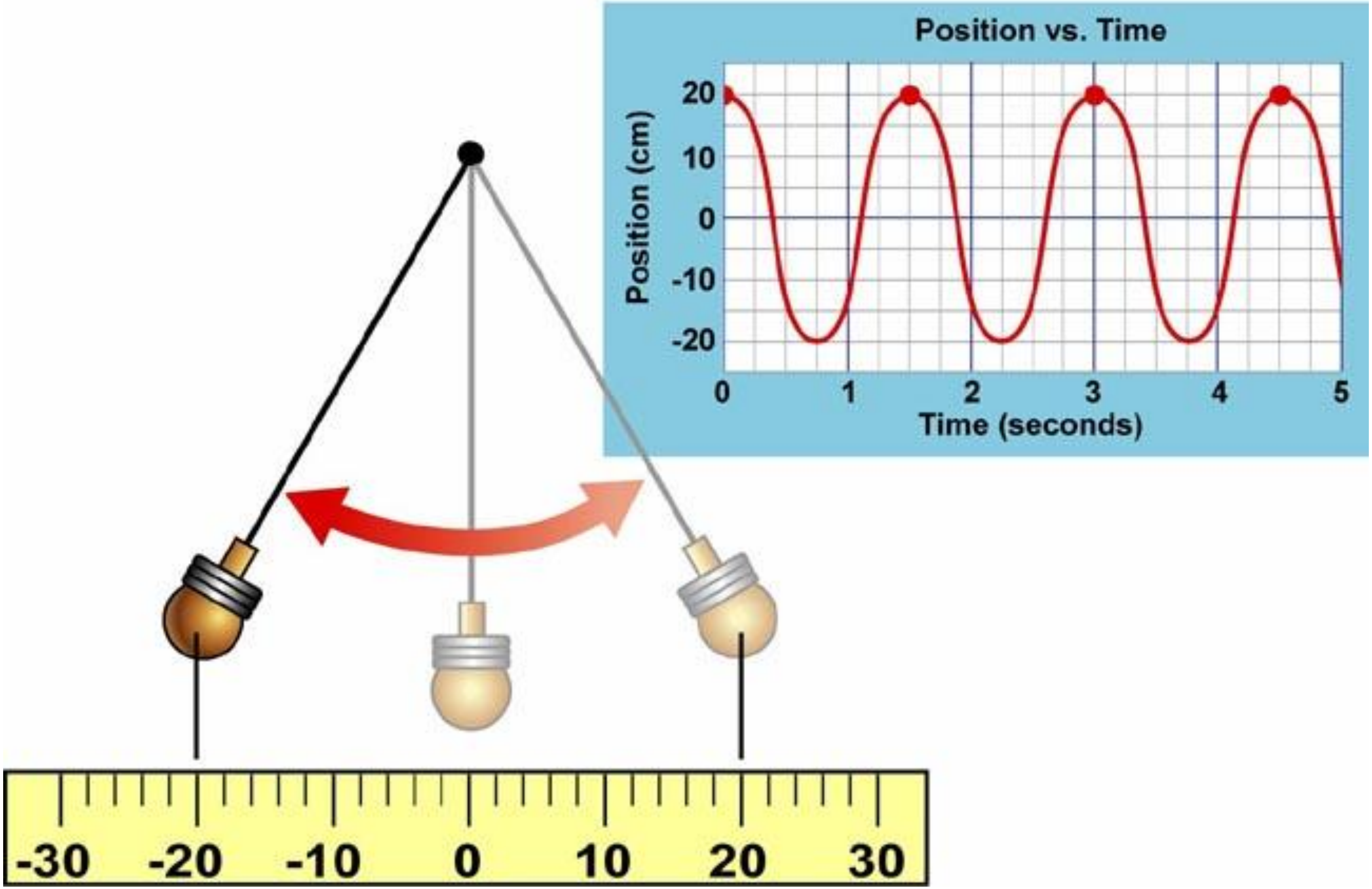
- Amplitude describes the size of a cycle.

The energy of an oscillator is proportional to the amplitude of the motion.



- **Friction drains يستنزف energy away from motion and slows the pendulum down.**
- **Damping is the term used to describe this loss.**

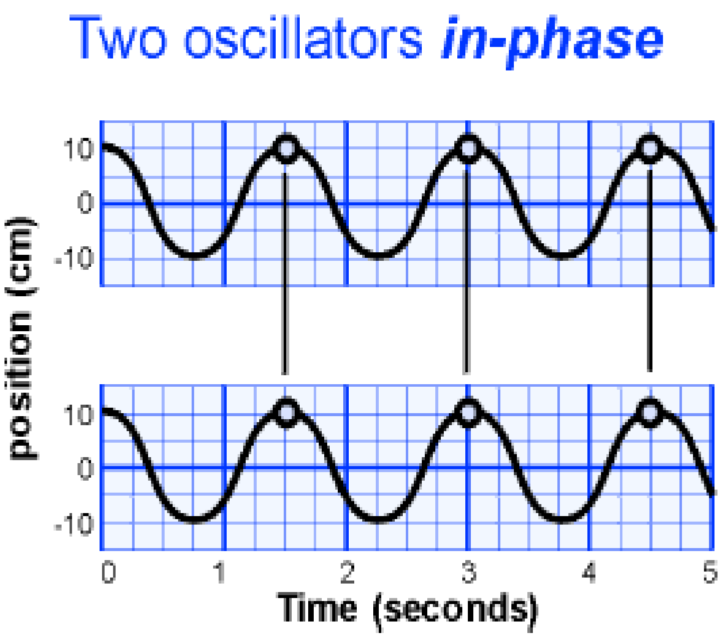
Harmonic Motion Graphs

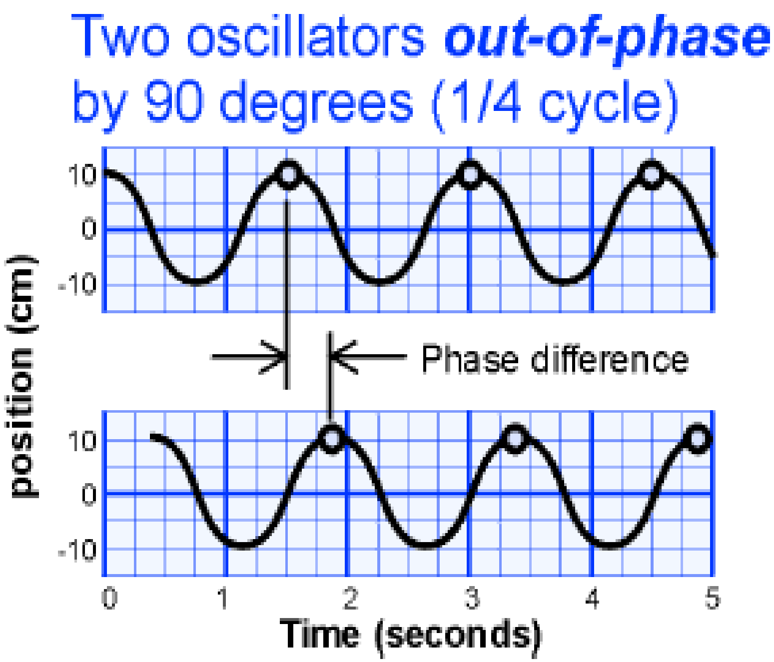


The phase of harmonic motion

The word "phase" means where the oscillator is in the cycle.

The concept of phase is important when comparing one oscillator with another.





Energy in the Simple Harmonic Oscillator

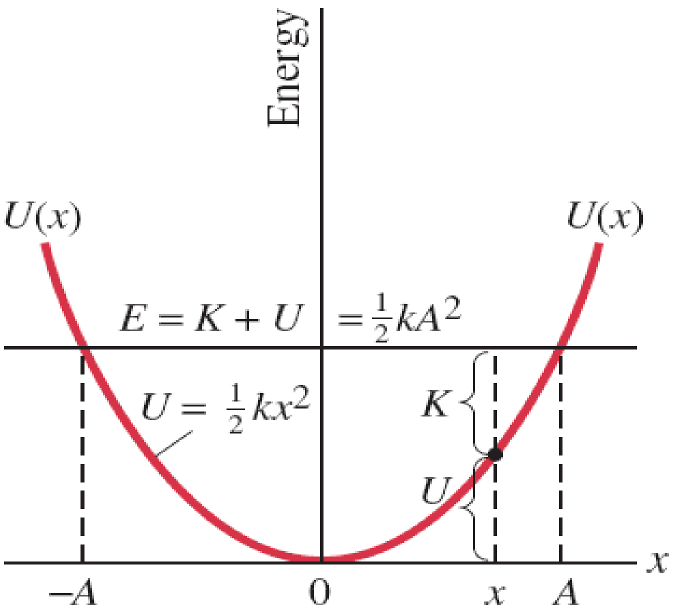
This graph shows the potential energy function of a spring. The total energy is constant.

The force acting on the particle at any position is derived from the potential energy function; its given by

F = - dU/dx

The total mechanical energy for an oscillsating is the sum of the potential energy and kinetic energy

, K + U = E = constant



, E = constant for any point *x*

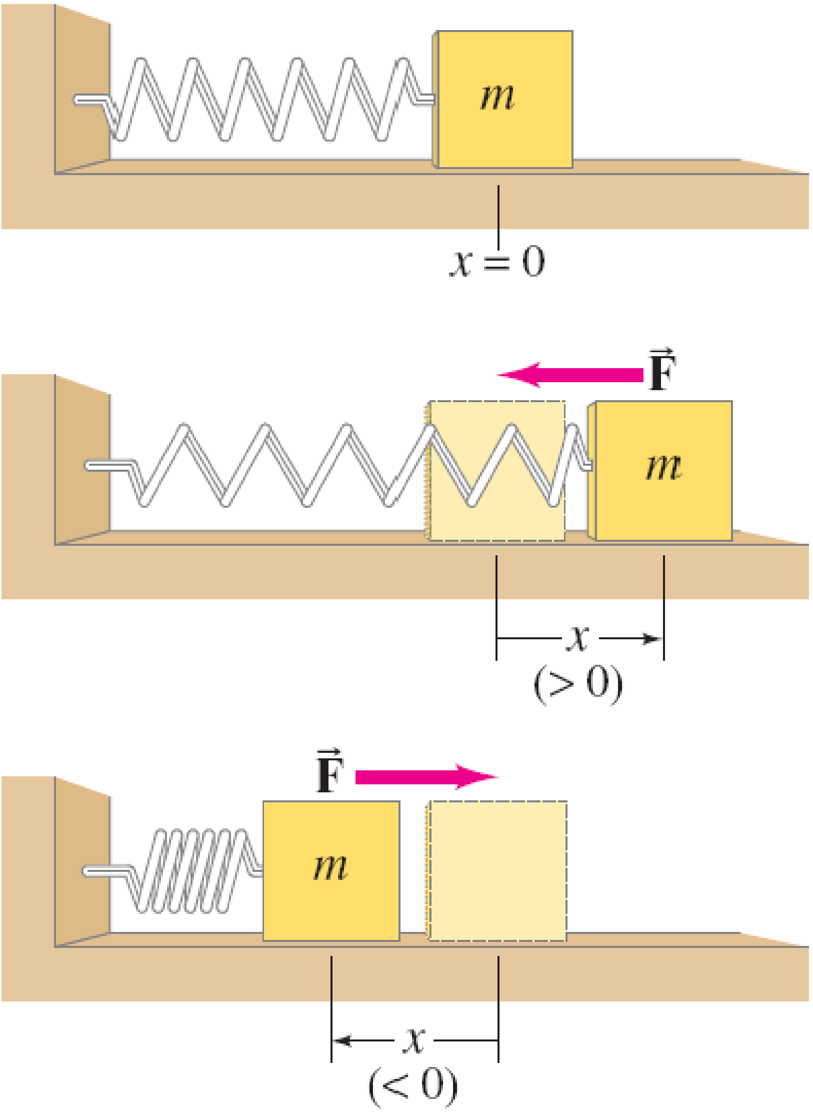
where –A ≤ x ≤ A.

Values of *K* and *U* are indicated for an arbitrary position *x*.

• Oscillations of a Spring

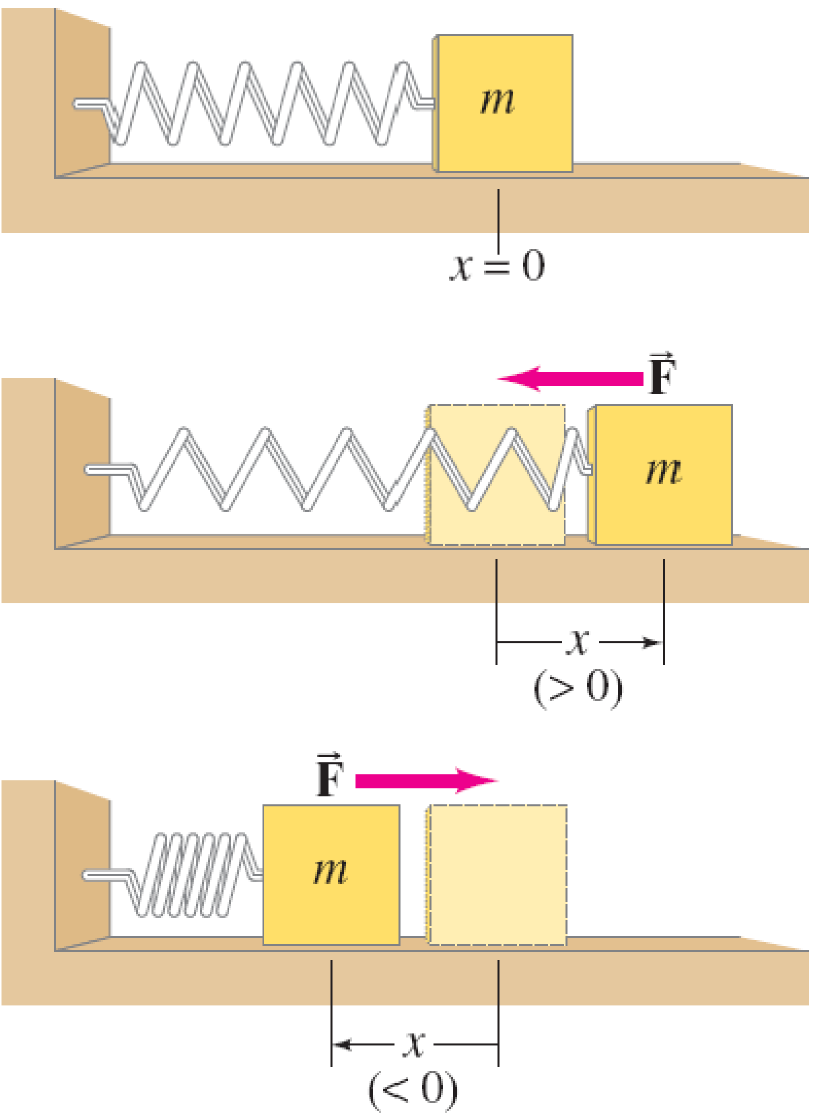
- Simple Harmonic Motion
- Energy in the Simple Harmonic Oscillator
- Simple Harmonic Motion Related to Uniform Circular Motion
- The Simple Pendulum
- The Physical Pendulum and the Torsion Pendulum
- Damped Harmonic Motion
- Forced Oscillations; Resonance

14-1 Oscillations of a Spring



If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic.

14-1 Oscillations of a Spring



We assume that the surface is **frictionless**.

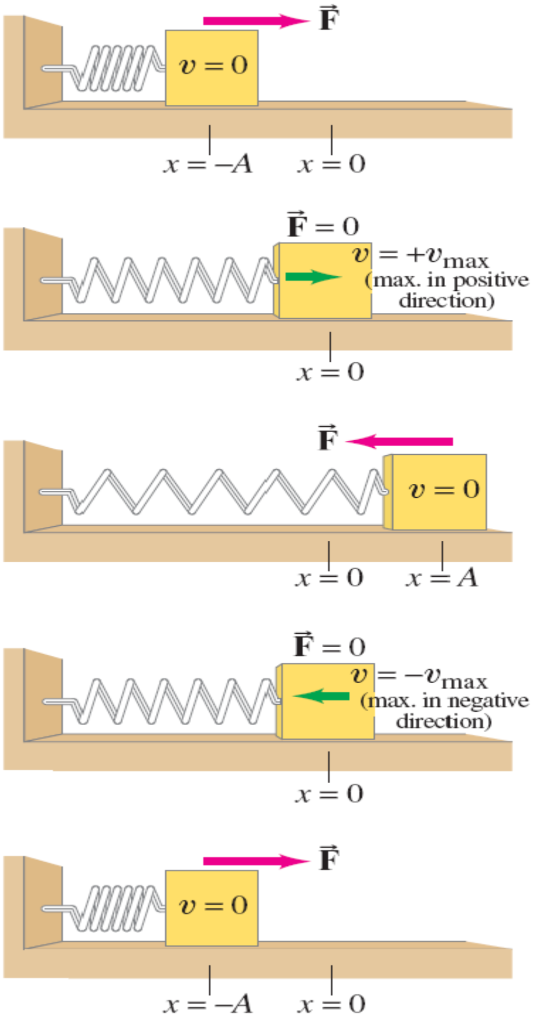
The **force** exerted by the spring depends on the **displacement**:

$$F = -kx.$$

Oscillations of a Spring

- $$F = -kx.$$
- Minus sign indicates a restoring force
  - *directed to restore the mass back to its equilibrium position.*
- k is the spring constant.
- Force is not constant, in magnitude nor direction, so acceleration is not constant either.

14-1 Oscillations of a Spring



- **Displacement** is measured from the equilibrium point.
- **Amplitude** is the maximum displacement.
- **A cycle** is a full to-and-fro motion.
- **Period** is the time required to complete one cycle.
- Frequency is the number of cycles completed per second.

Force on, and velocity of, a mass at different positions of its oscillation cycle on a frictionless surface

Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in ***simple harmonic motion*** *(****SHM****), and is often called a* ***simple harmonic oscillator*** *(****SHO****).*

Simple Harmonic Motion

Again, we know the force exerted by the spring is

$$F = -kx$$

And from Newton's second law

$$F = ma = m\,d^2x/dt^2 = -kx \qquad (1)$$

gives the general equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0, \qquad (2)$$

with solutions of the form:

$$x = A\cos(\omega t + \delta) \qquad (3)$$

The angular frequency

$$\boxed{\begin{array}{l} \omega = 2\pi f = \dfrac{2\pi}{T} \\ \text{SI unit: rad/s} = \text{s}^{-1} \end{array}}$$

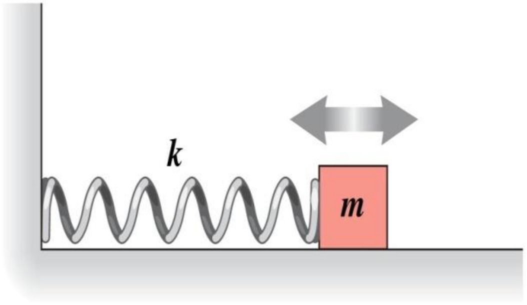
Simple Harmonic Motion, SHM

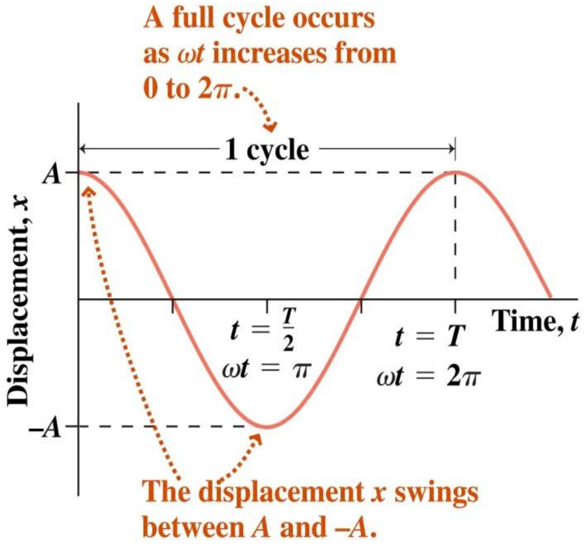
$$x = A\cos(\omega t + \delta)$$

and

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \delta), \qquad (4)$$

$$a = \frac{d^2x}{d^2t} = A\omega^2 \cos(\omega t + \delta) \qquad (5)$$





Therefore from Eq. 5, 7 and 8 we get

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$-A\omega^2 \cos(\omega t + \delta) = -\frac{k}{m}A\cos(\omega t + \delta) - - - (6)$$

Simple Harmonic Motion, Frequency and Period

From 9 We can set

$$m[-A\omega^2\cos(\omega t+\delta)] = -k[A\cos(\omega t+\delta)]$$

. `$k = m\, w^2$, **that is,**

$$\omega = \sqrt{\frac{k}{m}}$$

***By definition*, after a period** T,

later the motion repeats, therefore:

$$x = A\cos(\omega t+\delta)$$

is a *more* general solution of the equation of motion.

The symbol δ is called the phase. It defines the *initial displacement*

$x = A\cos\delta$

Simple Harmonic Motion, Frequency and Period

If the time t in equation $x = A\cos(\omega t + \delta)$ is increased by

$2\pi/\omega$, the function becomes

.

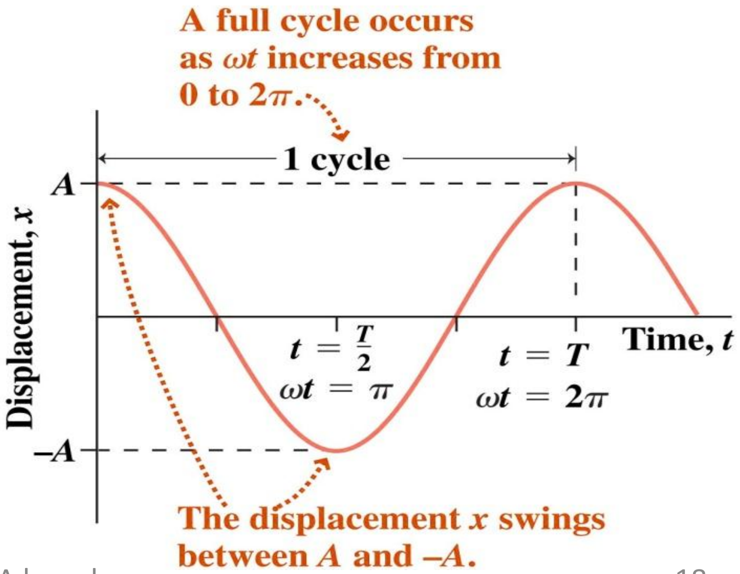
$$x = A\cos[(\omega t + 2\pi/\omega) + \delta)$$

That is the function merely repeats itself after a time $2\pi/\omega$. Therefore $2\pi/\omega$ is the period of the motion T .

Since $\omega^2 = k/m$ we have

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{k}}$$

ω is called the **angular frequency**



Simple Harmonic Motion, Frequency and Period

For simple harmonic motion of the mass-spring system, the frequency υ of oscillator is the number of complete vibrations per unit time and given by:

$$\upsilon = 1/T = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

And the frequency write

$$\omega = 2\pi\upsilon = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

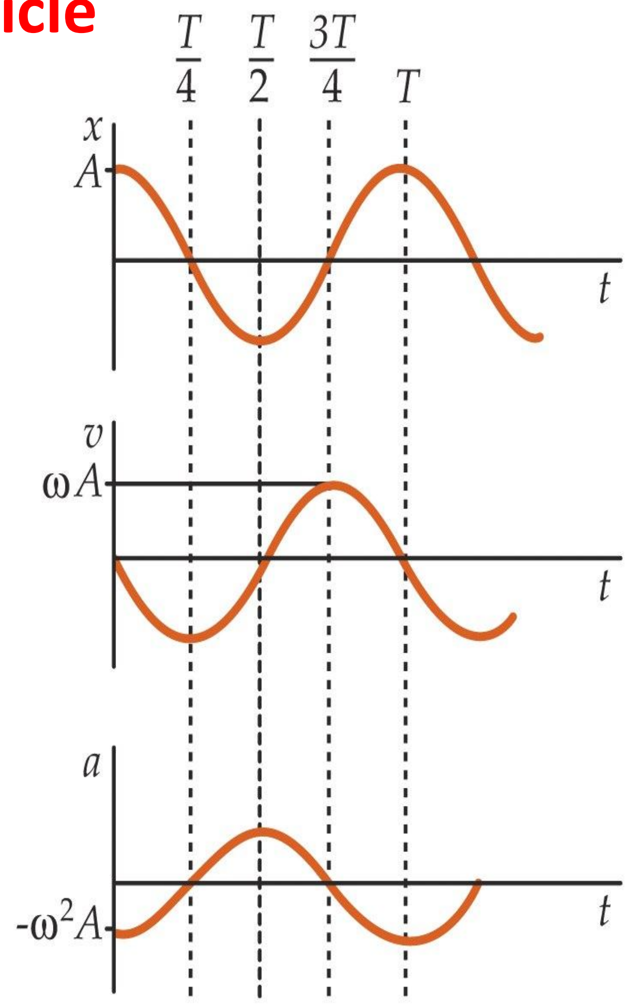
Simple Harmonic Motion - Position, Velocity, Acceleration

For SHM the relation between the **displacement , the Velocity, and the Acceleration of oscillation particle Is given by**

$$x = A\cos(\omega t + \delta)$$

$$V = \frac{dx}{dt} = -A\omega \sin(\omega t + \delta),$$

$$a = \frac{d^2x}{d^2t} = A\omega^2 \cos(\omega t + \delta)$$



Note that,

$X_{max} = A$,

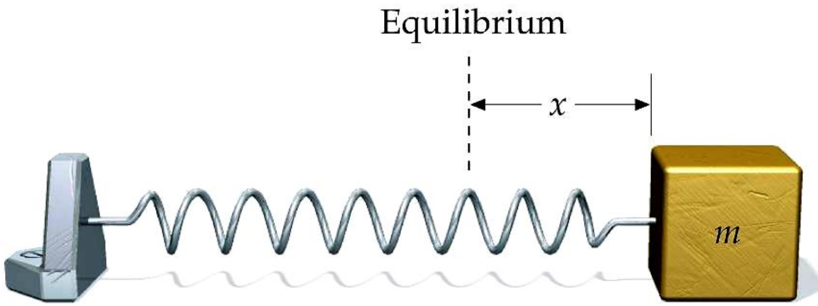
$V_{max} = A\omega$,

$a_{max} = A\ \omega^2$.

Mass+Spring
Simple Harmonic Motion

$$F = -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$



In simple harmonic motion (SHM), the acceleration, and thus the net force, are both proportional to and oppositely directed from the displacement from the equilibrium position.

$$\text{frequency} = f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$X(t) = A\cos(\omega t + \varphi)$$

$A =$ amplitude

$\omega =$ angular frequency

$\varphi =$ phase

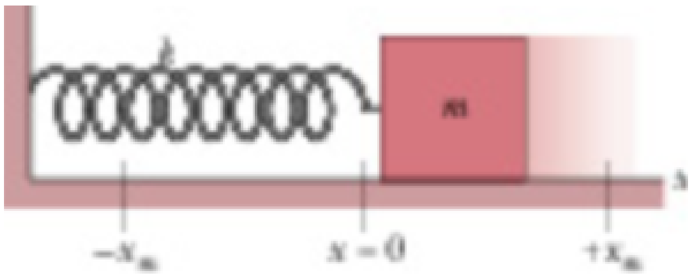
The Force Law for Simple SHM

$$F = ma = -(m\omega^2)x$$

$$F = -kx \text{ (Hooke's Law)}$$

$$k = m\omega^2 \text{ (for a spring)}$$

Simple Harmonic Motion is the motion executed by a particle of mass m subject to a force that is proportional to the displacement of the particle but opposite in sign



$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$

- **Note that**.
  - The Maximum displacement is
    - $X_{max} = A$
  - The Maximum velocity
    - $V_{max} = \omega A$
  - The maximum acceleration
    - $a_{max} = \omega^2 A$

Sample Problem 15-1

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance x = 11 cm from its equilibrium position x = 0 on a frictionless surface and released from rest at t = 0.(a) what are the angular frequency, the frequency, and the period of the resulting motion? (b) What is the amplitude of the oscillation? (c) What is the maximum speed v_m of the oscillating block, and where is the block when it occurs? (d) What is the magnitude a_m of the maximum acceleration of the block? (e) What is the phase constant Φ for the motion? (f) What is the displacement function x(t) for the spring block system?

a) $\omega = \sqrt{\dfrac{k}{m}} = 9.8 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 1.6 \text{ Hz}$$

$$T = \frac{1}{f} = 0.64 \text{ s}$$

b) $x_m = 11 \text{ cm}$

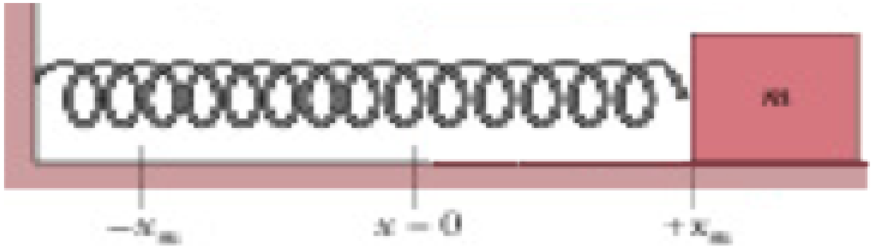
c) $v_m = \omega x_m = 1.1 \text{ m/s}$

d) $a_m = \omega^2 x_m = 11 \text{ m/s}^2$

e) at $t = 0$, $x = x_m \rightarrow$

$$x = x_m \cos(\omega t + \phi) \rightarrow 1 = \cos(\phi) \rightarrow \phi = 0$$

f) $x(t) = 0.11 \cos(9.8t)$



Sample Problem 15-2

At t=0, the displacement x(0) of the block in a linear oscillator is -8.50 cm. The block's velocity v(0)=-0.920 m/s, and the acceleration a(0)=+47.0 m/s^2

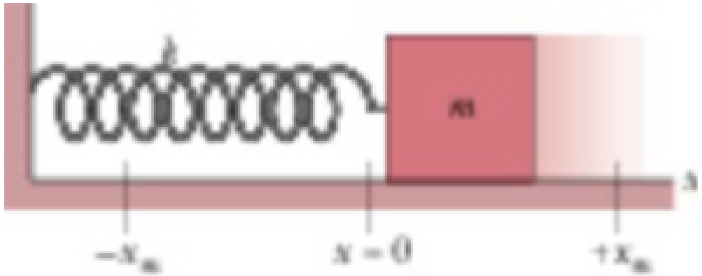
a) What is the angular frequency ω of this system?

b) What is the phase constant Φ and amplitude x_m?

a) $x(0) = x_m\cos(\phi), \quad v(0) = -\omega x_m\sin(\phi), \quad a(0) = -\omega^2 x_m\cos(\phi)$

$$\omega = \sqrt{-\frac{a(0)}{x(0)}} = 23.5\ rad/s$$

b) $\frac{v(0)}{x(0)} = -\omega\tan(\phi) \rightarrow \tan(\phi) = \frac{v(0)}{\omega x(0)} = -0.461$



$\phi = -25^o \quad and \quad \phi = 180^o + (-25^o) = 155^o$

$for \quad \phi = -25^o: \quad x_m = \frac{x(0)}{\cos(\phi)} = -0.094\ m$

$(\phi = -25^o \quad rejected \quad since \quad it \quad gives \quad \text{-}ve \quad amplitude)$

$for \quad \phi = 155^o: \quad x_m = \frac{x(0)}{\cos(\phi)} = +0.094\ m$

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Energy in Simple Harmonic Motion

For SHM the displacement is given by

$$x = A\cos(\omega t + \delta)$$

The total energy is given by $E = K + U$

The potential energy U at any instant is given by $\frac{1}{2} k x^2$

$$U = \tfrac{1}{2} kA^2 \cos^2 (\omega t + \delta)$$

The kinetic energy K at any instant is given by $\frac{1}{2} m V^2$

Velocity $V = dx/dt = -A\omega \sin (\omega t + \delta)$

$K_E = \frac{1}{2} mA^2 \omega^2 \sin^2 (\omega t + \delta)$, ***where for spring*** $\omega^2 = k/m$,

$$K_E = \tfrac{1}{2} kA^2 \sin^2 (\omega t + \delta)$$

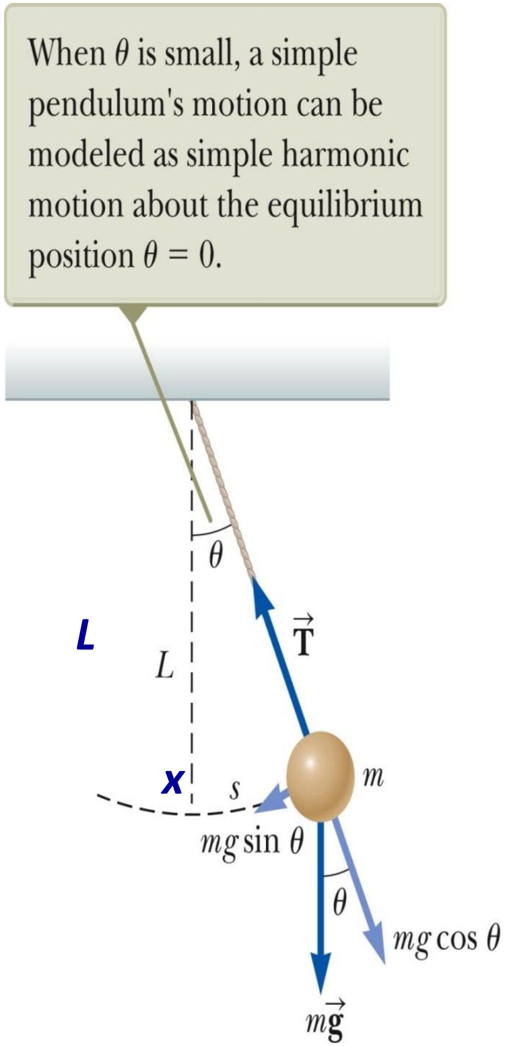
Then ,Total Energy = Kinetic Energy + Potential Energy

$$E = \tfrac{1}{2} kA^2 \sin^2 (\omega t + \delta) + \tfrac{1}{2} kA^2 \cos^2 (\omega t + \delta)$$

$$= \tfrac{1}{2} kA^2 (\sin^2 (\omega t + \delta) + \cos^2 (\omega t + \delta) = \tfrac{1}{2} kA^2$$

Simple Pendulum, SHM

- **The forces acting on the bob are the tension and the weight.**
- **T is the force exerted by the string mg is the gravitational force**
- **The tangential component of the restoring force acting on m tend to return to its equilibrium position . Hence the restoring force is.**
- **$F = -mg \sin\theta$**
- **If the angle θ is very small, then $\sin\theta = \theta$**
- **Then, $F = -mg\,\theta = -\,mg\,x/L = -\,(mg/L)\,x$**
- **For small displacements, the restoring force is proportional to oppositely direction of displacement. $F = -\,kx$**
- **The constant (mg/L) represents the constant k in**
- **, $F = -\,kx$ that is , $k = (mg/L)$**
- **The period of simple pendulum at small amplitude is**
- $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$$



Therefore, for small angles, we have:

- **The constant (mg/L) represents the constant k in**
- **, F = - kx that is , k = (mg/L)**
- **The period and frequency of simple pendulum at small amplitude are:**

$$F = -kx \quad \text{-----} \quad k = mg/L\,,$$

$$K = \omega^2 m$$

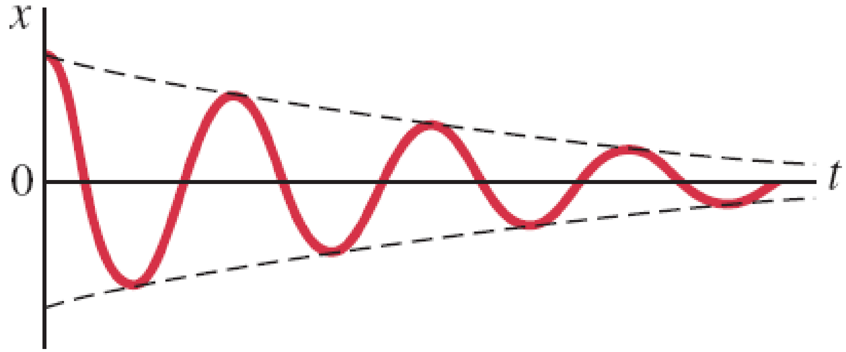
$$m(2\pi f)^2 = m\, g/L$$

$$f^2 = g/4\pi^2 L$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.} \qquad \boxed{T = 2\pi\sqrt{\frac{L}{g}},}$$

Damped Harmonic Motion

Damped harmonic motion is harmonic motion with a frictional or drag السحب force . If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.



If

$$F_{\mathrm{damping}} = -bv,$$

then

$$ma = -kx - bv.$$

This gives

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

If b is small, a solution of the form

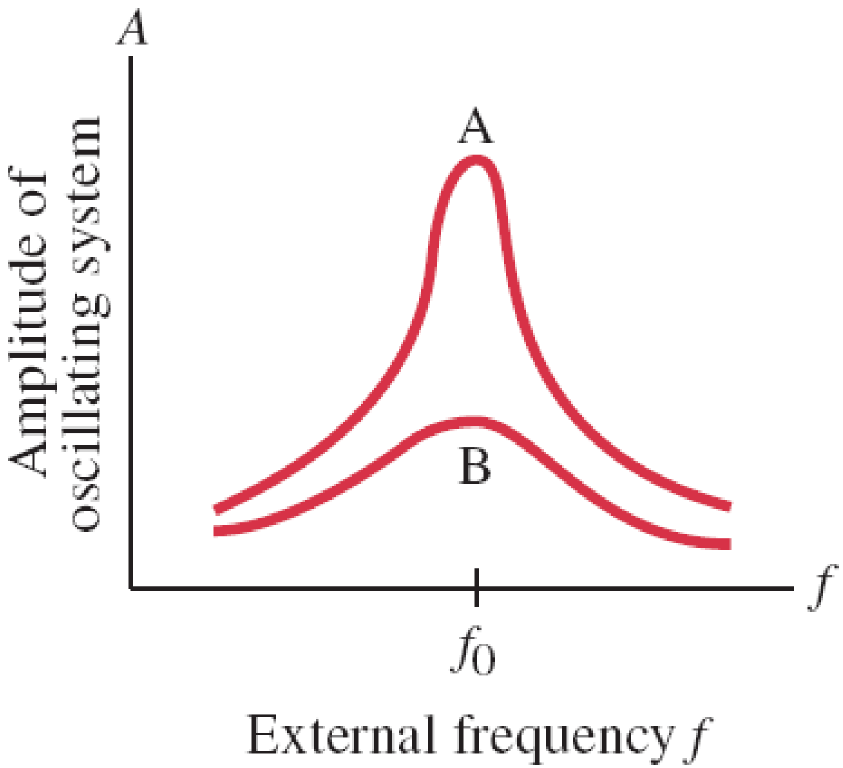
$$x = Ae^{-\gamma t}\cos\omega' t$$

will work, with

$$\gamma = \frac{b}{2m}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

Forced Oscillations; Resonance



The sharpness of the resonant peak depends on the damping. If the damping is small (A) it can be quite sharp; if the damping is larger (B) it is less sharp.

Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.

The equation of motion for a forced oscillator is:

$$ma = -kx - bv + F_0 \cos \omega t.$$

The solution is:

$$x = A_0 \sin(\omega t + \phi_0),$$

where

$$A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + b^2\omega^2/m^2}}$$

and

$$\phi_0 = \tan^{-1} \frac{\omega_0^2 - \omega^2}{\omega(b/m)}.$$

14-8 Forced Oscillations; Resonance

The width of the resonant peak can be characterized by the Q factor:

$$Q = \frac{m\omega_0}{b}.$$